РЕШЕНИЕ ТРИГОНОМЕТРИЧЕСКИХ УРАВНЕНИЙ
1 СПОСОБ. Уравнения, сводящиеся к квадратным

1) $\sin^2 x + \sin x - 2 = 0$

Пусть $\sin x = y$, $-1 \le y \le 1$

$y^2 + y - 2 = 0$

$a = 1;\ b = 1,\ c = -2$

$D = b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot (-2) = 9 = 3^2$

$$y_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm 3}{2 \cdot 1} = \frac{-1 \pm 3}{2}$$

$$y_1 = \frac{-1+3}{2} = \frac{2}{2} = 1$$

$$y_2 = \frac{-1-3}{2} = \frac{-4}{2} = -2 < -1 \text{ не подходит}$$

Итак, $y=1$. Значит,

$\sin x = 1;\ x = \frac{\pi}{2} + 2\pi n,\ n \in Z$

2) $2\cos^2 x - \cos x - 1 = 0$

Пусть $\cos x = y$, $-1 \le y \le 1$

$2y^2 - y - 1 = 0$

$a = 2,\ b = -1,\ c = -1$

$D = b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-1) = 9 = 3^2$

$$y_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm 3}{2 \cdot 2} = \frac{1 \pm 3}{4}$$

$$y_1 = \frac{1+3}{4} = \frac{4}{4} = 1$$

$$y_2 = \frac{1-3}{4} = \frac{-2}{4} = -\frac{1}{2}$$

Подходят оба значения корня

1) $\cos x = 1$

$x = 2\pi n,\ n \in Z$

2) $\cos x = -\frac{1}{2}$

$$x = \pm \arccos\left(-\frac{1}{2}\right) + 2\pi n$$

$$x = \pm\left(\pi - \arccos\frac{1}{2}\right) + 2\pi n$$

$$x = \pm\left(\pi - \frac{\pi}{3}\right) + 2\pi n$$

$$x = \pm\frac{2\pi}{3} + 2\pi n,\ n \in Z$$

3) $2\cos^2 x - 5\sin x + 1 = 0$ $\boxed{\cos^2 x = 1 - \sin^2 x}$

$2(1 - \sin^2 x) - 5\sin x + 1 = 0$

$2 - 2\sin^2 x - 5\sin x + 1 = 0$

$-2\sin^2 x - 5\sin x + 3 = 0 \mid \cdot (-1)$

$2\sin^2 x + 5\sin x - 3 = 0$

Пусть $\sin x = y$, $-1 \le y \le 1$

$2y^2 + 5y - 3 = 0$

$a = 2;\ b = 5;\ c = -3$

$D = b^2 - 4ac = 5^2 - 4 \cdot 2 \cdot (-3) = 49 = 7^2$

$$y_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm 7}{2 \cdot 2} = \frac{-5 \pm 7}{4}$$

$$y_1 = \frac{-5+7}{4} = \frac{2}{4} = \frac{1}{2}$$

$$y_2 = \frac{-5-7}{4} = \frac{-12}{4} = -3 < -1 \text{ не подходит}$$

Итак, $y = \frac{1}{2}$. Имеем:

$\sin x = \frac{1}{2}$

$x = (-1)^n \arcsin\frac{1}{2} + \pi n,\ n \in Z$

$x = (-1)^n \frac{\pi}{6} + \pi n,\ n \in Z$

4) $4\sin^2 x - \cos x - 1 = 0$ $\boxed{\sin^2 x = 1 - \cos^2 x}$

$4(1 - \cos^2 x) - \cos x - 1 = 0$

$4 - 4\cos^2 x - \cos x - 1 = 0$

$-4\cos^2 x - \cos x + 3 = 0 \mid \cdot (-1)$

$4\cos^2 x + \cos x - 3 = 0$

Пусть $\cos x = y$, $-1 \le y \le 1$

$4y^2 + y - 3 = 0$

$a = 4;\ b = 1;\ c = -3$

$D = b^2 - 4ac = 1^2 - 4 \cdot 4 \cdot (-3) = 49 = 7^2$

$$y_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm 7}{2 \cdot 4} = \frac{-1 \pm 7}{8}$$

$$y_1 = \frac{-1+7}{8} = \frac{6}{8} = \frac{3}{4}$$

$$y_2 = \frac{-1-7}{8} = \frac{-8}{8} = -1$$

Подходят оба значения корня

1) $\cos x = \frac{3}{4}$

$x = \pm \arccos\frac{3}{4} + 2\pi n$,

$n \in Z$

2) $\cos x = 1$

$x = \pi + 2\pi n,\ n \in Z$

2 СПОСОБ. Уравнения вида *a sin x + b cos x = c*

5) $2\sin x - 3\cos x = 0 \mid : \cos x$

$2\frac{\sin x}{\cos x} - 3\frac{\cos x}{\cos x} = 0$

$2tg\, x - 3 = 0$

$2tg\, x = 3$

$tg\, x = \frac{3}{2}$

$x = arctg\frac{3}{2} + \pi n,\ n \in Z$

6) $2\sin x + \cos x = 2$

$2\sin x + \cos x - 2 = 0$

$2\sin x + \cos x - 2 \cdot 1 = 0$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$
$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$$
$$1 = \cos^2\frac{x}{2} + \sin^2\frac{x}{2}$$

$$2 \cdot 2\sin\frac{x}{2}\cos\frac{x}{2} + \left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right) - 2 \cdot \left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) = 0$$

$$4\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2} - \sin^2\frac{x}{2} - 2\cos^2\frac{x}{2} - 2\sin^2\frac{x}{2} = 0$$

$$4\sin\frac{x}{2}\cos\frac{x}{2} - \cos^2\frac{x}{2} - 3\sin^2\frac{x}{2} = 0$$

$$-3\sin^2\frac{x}{2} + 4\sin\frac{x}{2}\cos\frac{x}{2} - \cos^2\frac{x}{2} = 0 \,\Big|\cdot(-1)$$

$$3\sin^2\frac{x}{2} - 4\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2} = 0 \,\Big|: \cos^2\frac{x}{2}$$

$$3tg^2\frac{x}{2} - 4tg\frac{x}{2} + 1 = 0$$

Пусть $tg\frac{x}{2} = y$

$3y^2 - 4y + 1 = 0$

$a = 3;\ b = -4;\ c = 1$

$D = b^2 - 4ac = (-4)^2 - 4 \cdot 3 \cdot 1 = 4 = 2^2$

$$y_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm 2}{2 \cdot 3} = \frac{4 \pm 2}{6}$$

$$y_1 = \frac{4+2}{6} = \frac{6}{6} = 1$$

$$y_2 = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}$$

1) $tg\frac{x}{2} = 1$

$\frac{x}{2} = arctg 1 + \pi n$

$\frac{x}{2} = \frac{\pi}{4} + \pi n \,\big|\cdot 2$

$x = \frac{\pi}{2} + 2\pi n, n \in Z$

2) $tg\frac{x}{2} = \frac{1}{3}$

$\frac{x}{2} = arctg\frac{1}{3} + \pi n \,\big|\cdot 2$

$x = 2arctg\frac{1}{3} + 2\pi n, n \in Z$

3 СПОСОБ. Уравнения, решаемые разложением левой части на множители

При решении уравнений этого вида используются формулы

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad (1)$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} \qquad (2)$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad (3)$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad (4)$$

7) $\sin 5x - \sin x = 0$

По формуле (2) имеем:

$$2\sin\frac{5x-x}{2}\cos\frac{5x+x}{2}=0$$

$$2\sin\frac{4x}{2}\cos\frac{6x}{2}=0$$

$$2\sin 2x\cos 3x=0$$

1) $\sin 2x = 0$

$2x = \pi n \mid : 2$

$x = \frac{\pi n}{2}, n\in Z$

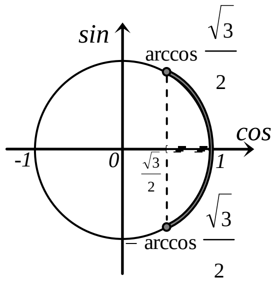
2) $\cos 3x = 0$

$3x=\frac{\pi}{2}+\pi n \mid : 3$

$x=\frac{\pi}{6}+\frac{\pi n}{3}, n\in Z$

РЕШЕНИЕ ТРИГОНОМЕТРИЧЕСКИХ НЕРАВЕНСТВ

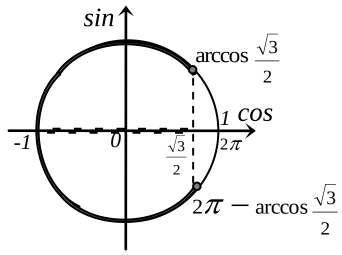
1) $\cos x > \dfrac{\sqrt{3}}{2}$



$$-\arccos\frac{\sqrt{3}}{2}+2\pi n < x < \arccos\frac{\sqrt{3}}{2}+2\pi n$$

$$-\frac{\pi}{6}+2\pi n < x < \frac{\pi}{6}+2\pi n, n \in Z$$

2) $\cos x \le \dfrac{\sqrt{3}}{2}$

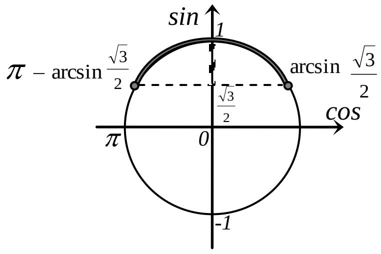


$$\arccos\frac{\sqrt{3}}{2}+2\pi n \le x \le 2\pi-\arccos\frac{\sqrt{3}}{2}+2\pi n$$

$$\frac{\pi}{6}+2\pi n \le x \le 2\pi-\frac{\pi}{6}+2\pi n$$

$$\frac{\pi}{6}+2\pi n \le x \le \frac{11\pi}{6}+2\pi n, n \in Z$$

3) $\sin x \ge \dfrac{\sqrt{3}}{2}$

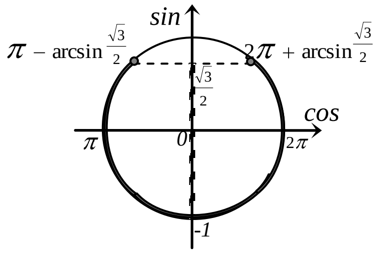


$$\arcsin\frac{\sqrt{3}}{2}+2\pi n \le x \le \pi-\arcsin\frac{\sqrt{3}}{2}+2\pi n$$

$$\frac{\pi}{3}+2\pi n \le x \le \pi-\frac{\pi}{3}+2\pi n$$

$$\frac{\pi}{3}+2\pi n \le x \le \frac{2\pi}{3}+2\pi n, n \in Z$$

4) $\sin x < \dfrac{\sqrt{3}}{2}$



$$\pi-\arcsin\frac{\sqrt{3}}{2}+2\pi n < x < 2\pi+\arcsin\frac{\sqrt{3}}{2}+2\pi n$$

$$\pi-\frac{\pi}{3}+2\pi n < x < 2\pi+\frac{\pi}{3}+2\pi n$$

$$\frac{2\pi}{3}+2\pi n < x < \frac{7\pi}{3}+2\pi n, n \in Z$$