Тригонометрические задачи со сложным аргументом.

Решить уравнения.

① $\sin(\underbrace{\sin(\cos x - \sin x)}_{t}) = 0$ $\quad \sin(\underbrace{\cos x - \sin x}_{z}) = \pi n,\ n \in Z$

$-1 \le \pi n \le 1 \Rightarrow n = 0 \quad \sin(\underbrace{\cos x - \sin x}_{z}) = 0$

$\cos x - \sin x = \pi k,\ k \in Z$

$\sqrt{2}\left(\cos x \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \sin x\right) = 1.$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{\pi k}{\sqrt{2}} \Rightarrow \sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}\,\pi k}{2};\ -1 \le \frac{\sqrt{2}\pi k}{2} \le 1$

$k = 0$

$\sin\left(x - \frac{\pi}{4}\right) = 0 \quad x - \frac{\pi}{4} = \pi n,\ n \in Z.\ x = \frac{\pi}{4} + \pi n,\ n \in Z.$

Ответ: $x = \frac{\pi}{4} + \pi n,\ n \in Z$

② $2\cos(\sqrt{x} + \pi) + 1 = 0 \quad \cos(\sqrt{x} + \pi) = -\frac{1}{2} \quad \cos\sqrt{x} = \frac{1}{2}$

$\sqrt{x} = \pm \arccos\frac{1}{2} + 2\pi k,\ k \in Z \quad \sqrt{x} \ge 0 \Rightarrow \sqrt{x} = \frac{\pi}{3} + 2\pi k,\ k \in Z$

$\sqrt{x} = -\frac{\pi}{3} + 2\pi k,\ k \in Z$

$x = \left(\frac{\pi}{3} + 2\pi k\right)^2 \quad x = \left(-\frac{\pi}{3} + 2\pi k\right)^2$

Ответ: $x = \left(\frac{\pi}{3} + 2\pi k\right)^2;\ x = \left(-\frac{\pi}{3} + 2\pi k\right)^2$

③ $\sin(2\pi \cos x) = 0$

$2\pi \cos x = \pi k,\ k \in Z,$

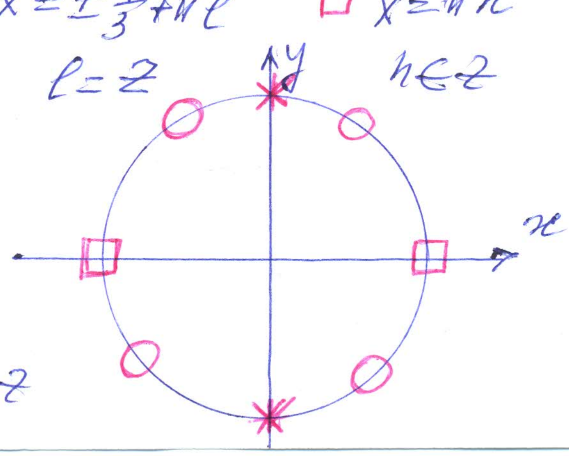
$\cos x = \frac{\pi k}{2\pi},\ \cos x = \frac{k}{2}\ k \in Z.$ т.к. $|\cos x| \le 1 \Rightarrow$

$-1 \le \frac{k}{2} \le 1 \ \Big| \cdot 2 \quad -2 \le k \le 2 \quad k = 0; \pm 1; \pm 2 \Rightarrow$

$\cos x = 0$ или $|\cos x| = \frac{1}{2}$ или $|\cos x| = 1$

$*\ x = \frac{\pi}{2} + \pi k,\ k \in Z \qquad \circ\ x = \pm\frac{\pi}{3} + \pi l,\ l \in Z \qquad \square\ x = \pi n,\ n \in Z$

Выборка корней



Ответ: $x_1 = \frac{\pi n}{2},\ n \in Z$

$x_2 = \pm\frac{\pi}{3} + 2\pi k,\ k \in Z$

④ $\underbrace{\cos\left(\frac{2}{3}\pi\sqrt{\cos x}\right)}_{z}=\frac{1}{2}$ $\quad \frac{2}{3}\pi\sqrt{\cos x}=\pm\frac{\pi}{3}+2\pi n,\ n\in Z \ \Big|\cdot\frac{3}{2\pi}$

$\sqrt{\cos x}=\pm\frac{\pi}{3}\cdot\frac{3}{2\pi}+2\pi n\cdot\frac{3}{2\pi},\ n\in Z$

$\sqrt{\cos x}=\pm\frac{1}{2}+3n,\ n\in Z.$

т. к. $0\le\sqrt{\cos x}\le 1$, то $0\le\pm\frac{1}{2}+3n\le 1 \Rightarrow$

$0\le\frac{1}{2}+3n\le 1$ или $0\le-\frac{1}{2}+3n\le 1$

$-\frac{1}{2}\le 3n\le 1-\frac{1}{2}$ $\qquad \frac{1}{2}\le 3n\le 1+\frac{1}{2}$

$-\frac{1}{2}\le 3n\le\frac{1}{2}\ \Big|\cdot\frac{1}{3}$ $\qquad \frac{1}{2}\le 3n\le\frac{3}{2}\ \Big|\cdot\frac{1}{3}$

$-\frac{1}{6}\le n\le\frac{1}{6}$ $\qquad \frac{1}{6}\le n\le\frac{1}{2}$ целых значений n нет.

$n=0.$

$\sqrt{\cos x}=\frac{1}{2}$ $\quad \cos x=\frac{1}{4}\Rightarrow x=\pm\arccos\frac{1}{4}+2\pi n$
$n\in Z$

Ответ: $x=\pm\arccos\frac{1}{4}+2\pi n,\ n\in Z$

⑤ $\underbrace{\cos\left(\pi\cdot\sqrt{1-\sin x}\right)}_{t}=1.$ $\quad \pi\cdot\sqrt{1-\sin x}=2\pi k,\ k\in Z.$

$\sqrt{1-\sin x}=\frac{2\pi k}{\pi}\quad k\in Z$

$\sqrt{1-\sin x}=2k$

$1-\sin x=4k^2\Rightarrow\sin x=1-4k^2.$ $\quad -1\le 1-4k^2\le 1$

$-2\le-4k^2\le 0$

$0\le k^2\le\frac{1}{2}\Rightarrow k=0.\Rightarrow$

$\sin x=1;\ x=\frac{\pi}{2}+2\pi n,\ n\in Z.$

Ответ: $x=\frac{\pi}{2}+2\pi k,\ k\in Z.$

⑥ $\sin^2(1-\cos x)=\cos^2(1+\cos x)$

используя формулы: $\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$ и

$\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$ получим

$\frac{1-\cos(2+2\cos x)}{2}-\frac{1+\cos(2-2\cos x)}{2}=0\ \Big|\cdot 2$

$\cos(2+2\cos\alpha)+\cos(2-2\cos x)=0$

$2\cos\frac{2+2\cos x+2-2\cos x}{2}\cdot\cos\frac{2+2\cos x-2+2\cos x}{2}=0$

$2\cos 2\cdot\cos(2\cos x)=0 \Rightarrow \cos(2\cos x)=0$

$2\cos x=\frac{\pi}{2}+\pi n,\ n\in Z \;\Big|\cdot\frac{1}{2}$

$\cos x=\frac{\pi}{4}+\frac{\pi n}{2},\ n\in Z$; $\quad -1\le\frac{\pi}{4}+\frac{\pi}{2}n\le 1$

$-1-\frac{\pi}{4}\le\frac{\pi}{2}n\le 1-\frac{\pi}{4}\;\Big|\cdot\frac{2}{\pi}$

$-\frac{2}{\pi}-\frac{1}{2}\le n\le\frac{2}{\pi}-\frac{1}{2}$; $n=0$ или $n=-1$

$-\frac{7}{6}\le n\le\frac{1}{6}$

$|\cos x|=\frac{\pi}{4}$ $\quad x=\pm\arccos\frac{\pi}{4}+\pi n,\ n\in Z$

Ответ: $x=\pm\arccos\frac{\pi}{4}+\pi n,\ n\in Z$

(7) $\sin\underbrace{\frac{\pi\cos x}{\cos^2 x+1}}_{t}=0$; $\quad \frac{\pi\cos x}{\cos^2 x+1}=\pi k,\ k\in Z$

$\pi\cos x=\pi k(\cos^2 x+1)\Rightarrow$

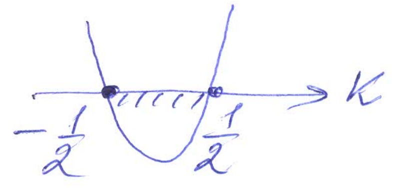
$\pi k\cos^2 x-\pi\cos x+\pi k=0$ квадратное ур-ие относительно $\cos x$.

Пусть $\cos x=t$, тогда $\underbrace{\pi k}_{a}t^2-\underbrace{\pi}_{b}t+\underbrace{\pi k}_{c}=0$

$D=\pi^2-4\pi^2k^2\ge 0$

$\pi^2-4\pi^2k^2\ge 0 \;|:\pi^2$

$1-4k^2\ge 0 \qquad 4k^2-1\le 0$



$-\frac{1}{2}\le k\le\frac{1}{2}\Rightarrow k=0$. $\quad \cos x=0$.

$x=\frac{\pi}{2}+\pi n,\ n\in Z$.

Ответ: $x=\frac{\pi}{2}+\pi n,\ n\in Z$

(8) $\sin\underbrace{\frac{2}{\operatorname{tg}x+\operatorname{ctg}x}}_{z}=\frac{1}{2}\Rightarrow\frac{2}{\operatorname{tg}x+\operatorname{ctg}x}=(-1)^k\cdot\arcsin\frac{1}{2}+\pi k$, $k\in Z$

$\frac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}=(-1)^k\frac{\pi}{6}+\pi k$, $k\in Z$ $\qquad 2\sin x\cdot\cos x=(-1)^k\frac{\pi}{6}+\pi k$, $k\in Z$.

$|\sin 2x|\le 1$

$-1 \le (-1)^k \frac{\pi}{6} + \pi k \le 1$

$-1-(-1)^k\frac{\pi}{6} \le \pi k \le 1-(-1)^k\frac{\pi}{6} \quad \Big| \cdot \frac{1}{\pi}$

$-\frac{1}{\pi}-(-1)^k\frac{1}{6} \le k \le \frac{1}{\pi}-(-1)^k\frac{1}{6}$ $\quad k=0. \quad \sin 2x = (-1)^0\frac{\pi}{6}+\pi\cdot 0$

$\sin 2x = \frac{\pi}{6} \quad 2x = (-1)^k \arcsin\frac{\pi}{6} + \pi n, n \in Z$

$x = (-1)^k \frac{1}{2} \arcsin\frac{\pi}{6} + \frac{\pi n}{2}; \ n \in Z$

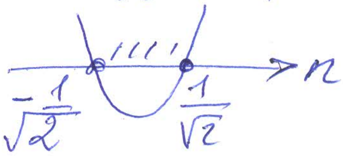
Ответ: $x = (-1)^n \frac{1}{2} \arcsin\frac{\pi}{6} + \frac{\pi n}{2}, \ n \in Z.$

9) $\cos \underbrace{\pi\sqrt{2-x^2}}_{t} = 1 \quad \pi\sqrt{2-x^2} = 2\pi n, \ n \in Z$

$\sqrt{2-x^2} = \frac{2\pi n}{\pi}; \quad \sqrt{2-x^2} = 2n \quad 2-x^2 = 4n^2$

$x^2 = 2-4n^2$, т.к. $x^2 \ge 0$, то $2-4n^2 \ge 0$.

$4n^2-2 \le 0 \quad \Big| \cdot \frac{1}{4}. \quad n^2 - \frac{1}{2} \le 0 \Rightarrow$



т.к $n \in Z$, то $n = 0 \Rightarrow$

$2 - x^2 = 0 \Rightarrow x^2 = 2. \quad x = \pm\sqrt{2}$

Ответ: $x = \pm\sqrt{2}$.